Convergence in High Probability of the Quantum Diffusion in a Random Band Matrix Model

In just 4 slides

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Introduction of the Model

In order to define the random matrices H with band width W, let us first consider

$$S_{xy} := \frac{\mathbf{1}(1 \le |x-y| \le W)}{M-1}.$$

We define the random band matrix (H_{xy}) through

$$H_{xy} := \sqrt{S_{xy}}\, A_{xy},$$

where (A_{xy}) is a Hermitian random matrix whose upper triangular entries $(A_{xy} : x \le y)$ are independent random variables uniformly distributed on the unit circle $S^1 \subset \mathbb{C}$. Note that the entries H_{xy} in the random band matrix H are indexed by x and y, which are indices of points in Λ_N (cube of size N in $\mathbb{Z}^d$).

Introduction of the Model

Let us consider also the function

$$P(t,x) = \left|(e^{-itH/2})_{0x}\right|^2,$$

that describes the quantum transition probability of a particle starting in 0 and ending in position x after time t.
For $\kappa > 0$, we introduce the macroscopic time and space coordinates T and X, which are independent of W, and consider the microscopic time and space coordinates

$$t = W^{d\kappa} T, \qquad x = W^{1+d\kappa/2} X.$$

We study:

$$Y_{T,\kappa,W}(\phi) \equiv Y_T(\phi) := \sum_x P\Big(W^{d\kappa} T, x\Big)\ \phi\Big(\frac{x}{W^{1+d\kappa/2}}\Big).$$

Expansion

Plugging in the definition of $Y_T(\phi)$ we have

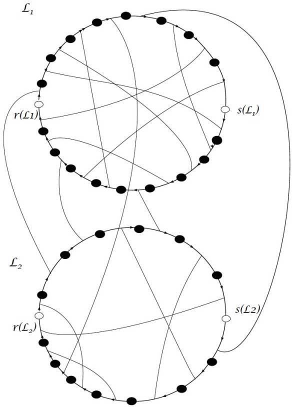
$$\mathrm{Var}(Y_T(\phi)) = \sum_{y_1,y_2} \phi\Big(\frac{y_1}{W^{1+d\kappa/2}}\Big)\,\phi\Big(\frac{y_2}{W^{1+d\kappa/2}}\Big)\,\langle P(t,y_1); P(t,y_2)\rangle$$

Moreover,

$$\begin{aligned}&\langle P(t,y_1); P(t,y_2)\rangle\\ &= \sum_{n_{11},n_{12}\geq 0}\ \sum_{n_{21},n_{22}\geq 0} a_{n_{11}}(t)a_{n_{12}}(t)a_{n_{21}}(t)a_{n_{22}}(t)\left\langle H_{0y_1}^{(n_{11})}H_{y_1 0}^{(n_{12})}\,;H_{0y_2}^{(n_{21})}H_{y_2 0}^{(n_{22})}\right\rangle. \qquad (1)\end{aligned}$$

Main result (M.) is a control of the $\mathrm{Var}(Y_T(\phi))$ and obtain the convergence in high probability via Chebyshev Inequality.

To control these terms, we use estimates based on graphical representations



- **What can we learn?** We analyze a model of Quantum Diffusion using Random Band Matrices and we show how one can use graphical representations based on certain Feynman graphs to obtain estimates in the analysis.